In the name of Allah,

Most Gracious, Most Merciful.

Article Title :

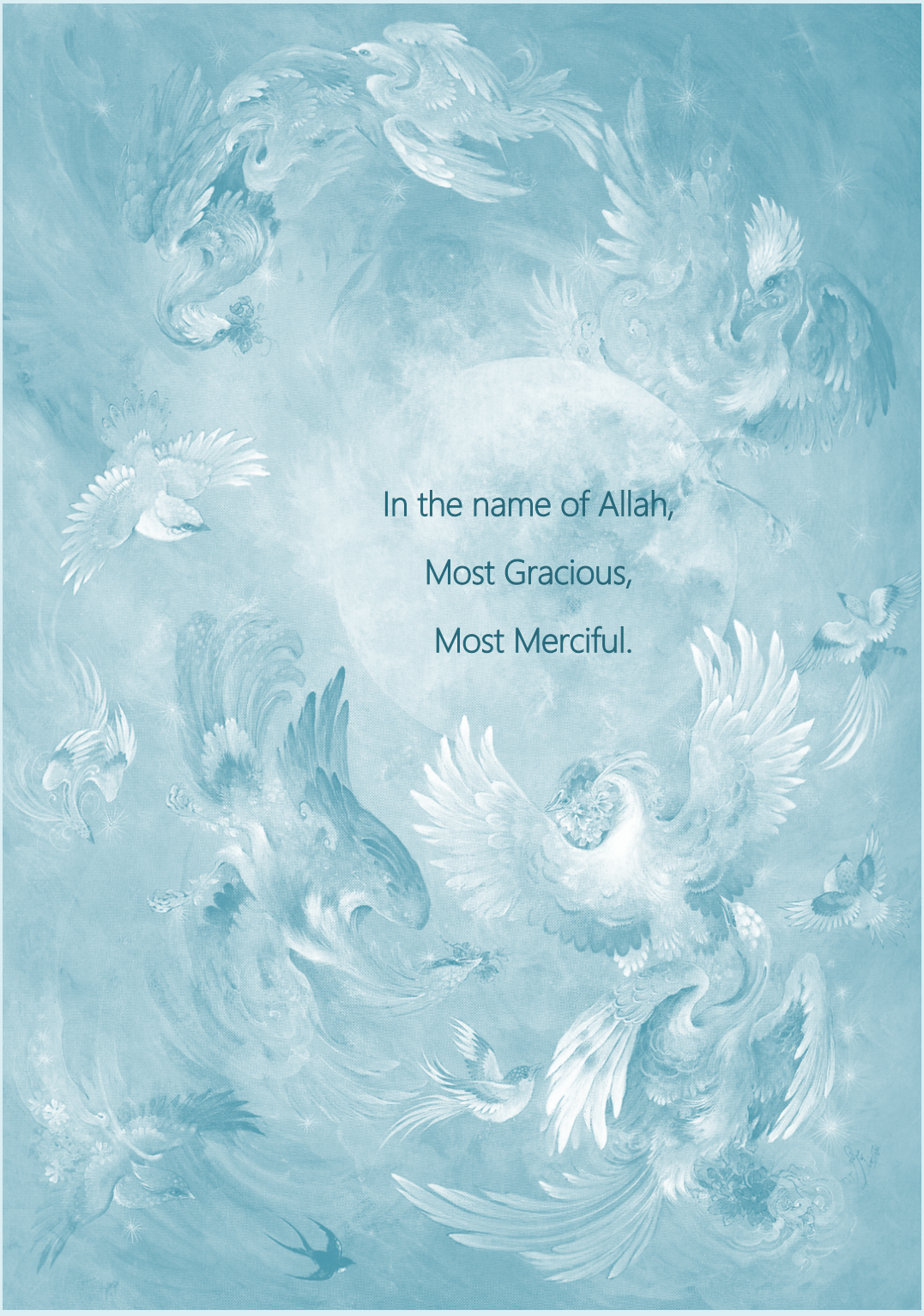
Miniatoric Infinity Plot And It,s Applications

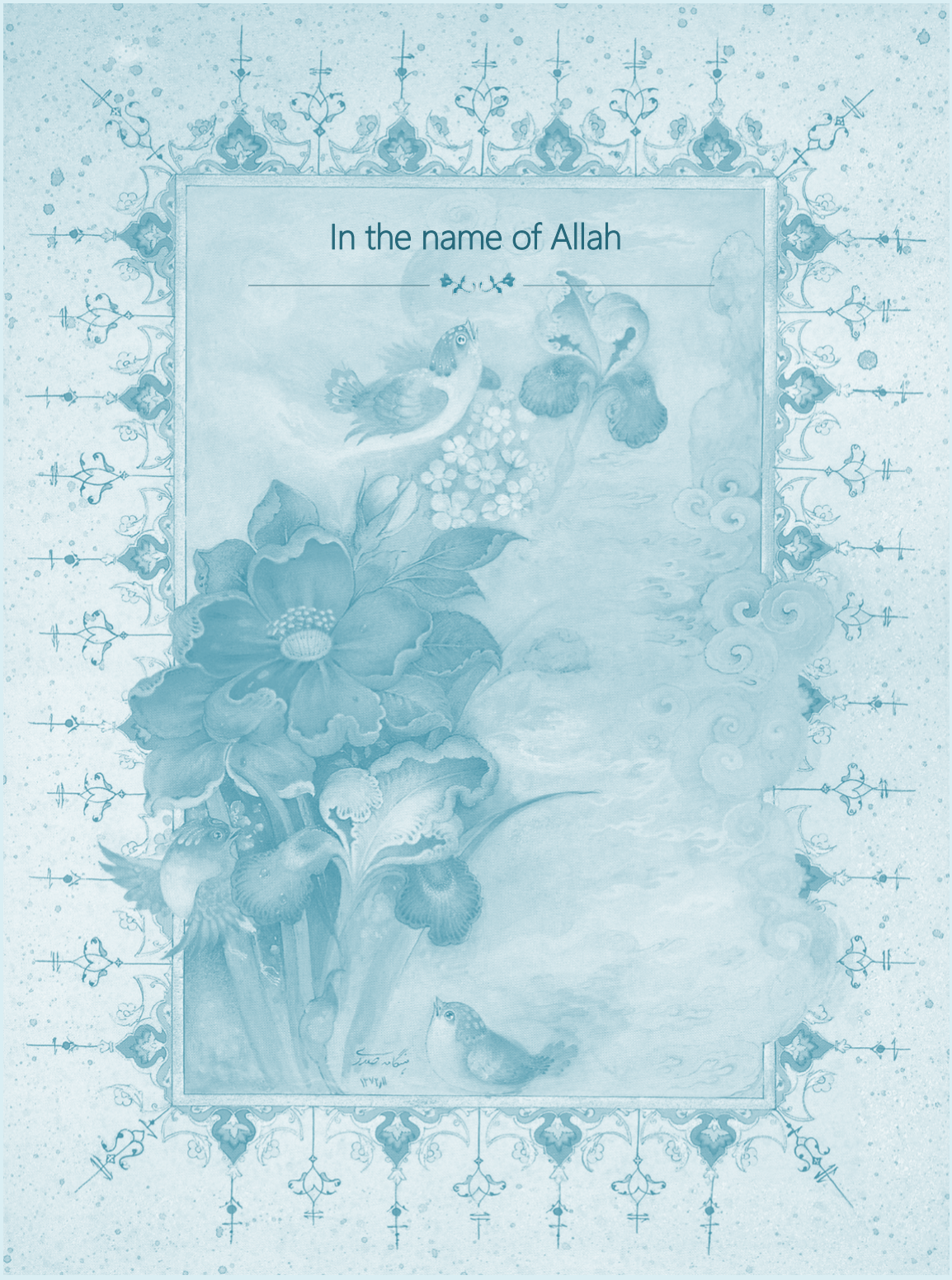
Digest Description Of Miniatoric Infinity Plot Concept :

- If either of the range end points of the horizontal range contains ±infinity, an infinity plot is generated.
- An infinity plot is obtained by transforming -infinity .. +infinity to $-\frac{\pi}{2}..\frac{\pi}{2}$ by a transformation that approximates arctan. This is a nice way of getting the entire picture of f(x) on the display.
- Such a graph, although distorted near x = -infinity and +infinity, contains a lot of information about the features of f(x).

The Article Is A Gift
From The GOD
To All The Believers In GOD

Do Not Forget The Ordained Prayer To GOD
Because Of This Great Manifest Signs Of The GOD.


In the name of Allah,
Most Gracious,
Most Merciful.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INFINITY PLOT APPLICATIONS

- ***Bisect Ghofrani Method to find roots***

*The function **Bisect** computes the root of a function within a given interval via the Bisection method. This is the simplest (and the slowest) method for root finding. However, its convergence is always guaranteed, and it is often used by more complex numerical methods. The algorithm implemented in Bisect is a modification of the ''classic'' algorithm, sometimes called ''Simplified Bisection'', and it is slightly faster.*

Syntax

```
root,error = Bisect(ftrans,xl,xr,eps,maxit,show)
```

Returns the roots of the function **ftrans** that uses the transformed type of the function **f** transformed with the following formula:

```
ArcTan(f(Tan(x))
```

inside the interval **{xl,xr}** that can be within the interval **{-$\frac{\pi}{2}$, $\frac{\pi}{2}$}**. The arguments **eps , maxit** and **show** are optional. **eps** defines the desired accuracy (default: **Epsilon**, i.e., 1.12×10^{-16}); it can be set to ''auto'', which is equivalent to the default accuracy. **maxit** defines the maximum number of iterations (default: 100). **show** is a boolean argument that controls wether progress of the iterative process will be displayed or not (default: **false**); if set to **true**, the function value, **f(root)**, at each iteration will be displayed.

Example

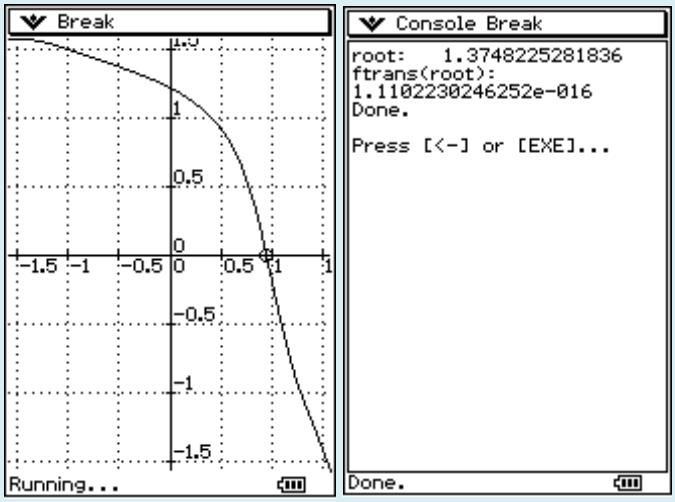
The example program **XBisGh** uses the function **Bisect** to find the root of the transformed function math.atan(f(math.tan(x))) where f is the function $\boldsymbol{x} - \frac{\mathbf{1}}{\mathbf{2}}\cos(\boldsymbol{x})$ within the interval **[-$\frac{\pi}{2}$, $\frac{\pi}{2}$]**. In order to check the result, the program computes the function value at the root ontained, and shows the result graphically.

```
require("LNAplot/PlotFunc","LNAplot/PlotData","LNA/Bisect")

local function f(x) return x-math.cos(x)/2 end
```

```
local function ftrans(x) return math.atan(f(math.tan(x))) end

local root,error,axesdata

root,error=Bisect(ftrans,-math.pi/2,math.pi/2)
print("root:",math.tan(root))
print("Estimated error:",error)
print("f(root):",f(math.tan(root)))

axesdata=PlotFunc(ftrans,{-math.pi/2,math.pi/2},{-math.pi/2,math.pi/2},false,1,"auto",true,1)
PlotData({root},{ftrans(root)},{},axesdata,true,2)
```

The Figure shows the results obtained by running this program. Note that function value **f(math.tan(root))**,is exactly zero in this particular example. This is rather the exception that the rule; in most cases, you should expect that **f(math.tan(root))** will be too close (but not equal) to zero.

Example Program : **XBisGH**



Results obtained by the example program **XBisGH**

Remarks

The interval **{xl,xr}** should contain *exactly* one root. If it does not, or if you need more that one root, you should use the function **KroneRoots** instead.

In Special cases, you may need to reduce the desired accuracy, using the optional argument **eps**. Usually, the maximum number of iterations, **maxit**, does not need to be changed; if the desired accuracy cannot be achieved with the default number of iterations, it is more than

likely that the desired accuracy is too high, and cannot be achieved by the Bisection method, despite the number of iterations.
FILENAME: **Bisect**.
DEPENDENCIES: **Epsilon**.

The Driver program code and it,s requirements code:

LNA/Bisect

```
require("LNAutils/Epsilon")


function Bisect(f,xl,xr,...)
local eps=Epsilon
local maxit=100
local show=false
if arg["n"]>=1 then
 if arg[1]~="auto" then
 eps=arg[1] end
 if arg["n"]>=2 then
  maxit=arg[2]
  if arg["n"]>=3 then
   show=arg[3]
  end
 end
end
```

```
local root,error
local xp,xc,fc,sl,sp,half
xp=xl;xc=xl
sl=math.sign(1,f(xl))
sp=sl;half=xr-xl
if sl*math.sign(1,f(xr))>0 then
 print("Error: Bisect: Interval does not contain a root, or contains an even number of roots.")
 return
end
if show then print(" i  discrepancy") end
for i=1,maxit do
 half=0.5*half
 xc=xp+sl*sp*half
 fc=f(xc)
 if show then printf("%2i  %+e\n",i,fc) end
 if half<=eps then
  root=xc;error=half
  return root,error
 end
 if fc==0 then
  root=xc;error=0
  return root,error
 else
```

```
 xp=xc
 sp=math.sign(1,fc)
 end
end
root=xc;error=half
print("Warning: Bisect: Iterations limit reached, but the accuracy is not satisfied.")
return root,error
end

export{Bisect=Bisect}
```

LNAutils/Epsilon

```
Epsilon=1.12E-16
export{Epsilon=Epsilon}
```

The Example program code and it,s requirements code:

LNAutils/XBisGh

```
require("LNAplot/PlotFunc","LNAplot/PlotData","LNA/Bisect")

local function f(x) return x-math.cos(x)/2 end

local function ftrans(x) return math.atan(f(math.tan(x))) end
```

```
local root,error,axesdata


root,error=Bisect(ftrans,-math.pi/2,math.pi/2)

print("root:",math.tan(root))

print("Estimated error:",error)

print("f(root):",f(math.tan(root)))


axesdata=PlotFunc(ftrans,{-math.pi/2,math.pi/2},{-math.pi/2,math.pi/2},false,1,"auto",true,1)

PlotData({root},{ftrans(root)},{},axesdata,true,2)
```

LNAutils/PlotFunc

```
require ("table","draw","LNAplot/PlotUtil")


function PlotFunc(f,xv,yv,...)

--Optional arguments:

local wait=true

local lwidth={1}

local tics={"auto","auto"}

local grid=true

local lblpos={0,0}

local lblsize={9,9}

local c="auto"
```

```
local discont={}
--
local scale
local lws,funcs,x,xp,X,Xp,Y,Yp,disconts
local id={}
if arg["n"]>=1 then
 wait=arg[1]
 if arg["n"]>=2 then
  lwidth=arg[2]
  if type(lwidth)=="number" then
   lwidth={lwidth}
  end
  if arg["n"]>=3 then
   tics=arg[3]
   if type(tics)~="table" then
    tics={tics,tics}
   end
   if arg["n"]>=4 then
    grid=arg[4]
    if arg["n"]>=5 then
     lblpos=arg[5]
     if type(lblpos)=="number" then
      lblpos={lblpos,lblpos}
```

```
    end
    if arg["n"]>=6 then
     lblsize=arg[6]
     if type(lblsize)=="number" then
      lblsize={lblsize,lblsize}
     end
     if arg["n"]>=7 then
      c=arg[7]
      if arg["n"]>=8 then
       discont=arg[8]
      end
     end
    end
   end
  end
 end
end
lws=#lwidth
if type(f)=="function" then
 f={f}
end
funcs=#f
```

```
for i=lws+1,funcs do
 lwidth[i]=lwidth[lws]
end
discont,disconts=Discontinuities(funcs,discont)
for i=1,funcs do
 id[i]=1
 table.sort(discont[i])
end
draw.onbuffer()
if xv[1]~=nil then
 scale=SetScale(xv,yv)
 PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)
else
 xv=yv[1]
 scale=yv[3]
 yv=yv[2]
end
Xp=0;xp=xv[1]
Y={};Yp={}
for i=1,funcs do
 Yp[i]=(yv[2]-f[i](xv[1]))*scale[2]
end
for X=1,X_pixels do
```

```
 x=xv[1]+X/scale[1]
 for i=1,funcs do
  Y[i]=(yv[2]-f[i](x))*scale[2]
  if id[i]>disconts[i] or x<discont[i][id[i]] or xp>discont[i][id[i]] then
   draw.line(Xp,Yp[i],X,Y[i],1,lwidth[i])
  elseif xp<=discont[i][id[i]] and x>=discont[i][id[i]] then
   id[i]=id[i]+1
  end
  Yp[i]=Y[i]
 end
 xp=x;Xp=X
end
showgraph()
draw.update()
if wait then waitkey() end
showconsole()
return {xv,yv,scale}
end


export{PlotFunc=PlotFunc}
```

LNAutils/PlotUtil

```
require ("string","table","draw","LNAutils/OrderMag")


X_max=158

Y_max=213

X_center=79

Y_center=106

X_pixels=159

Y_pixels=214


local function SetScale(xv,yv)

local xscale,yscale

xscale=X_pixels/(xv[2]-xv[1])

yscale=Y_pixels/(yv[2]-yv[1])

return {xscale,yscale}

end


local function AutoXtics(range)

--tics=math.max(math.floor(range/7),1)

local tics=OrderMag(range)

if range<4*tics then

 tics=tics/2

end

return tics
```

```
end


local function AutoYtics(range)
--tics=math.max(math.floor((yv[2]-yv[1])/9),1)
local tics=OrderMag(range)
if range<4*tics then
 tics=tics/2
end
return tics
end


function Discontinuities(funcs,discont)
local D=table.copy(discont)
local id={}
local disconts={}
for i=1,funcs do
 if D[i]==nil then
  D[i]={}
 elseif type(D[i])=="number" then
  D[i]={D[i]}
 end
 disconts[i]=#D[i]
end
```

```
return D,disconts

end


function PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)

local Xc,Yc,X,Y,Xl,Yl,lblxp,lblyp

local lblcut=1E-8

--Auto tics selection:

if tics[1]=="auto" then

 tics[1]=AutoXtics(xv[2]-xv[1])

end

if tics[2]=="auto" then

 tics[2]=AutoYtics(yv[2]-yv[1])

end

--

if c=="auto" then

 c={0,0}

end

--[[ AXES ]]--

Xc=(c[1]-xv[1])*scale[1]

Yc=(yv[2]-c[2])*scale[2]

draw.line(0,Yc,X_max,Yc)

draw.line(Xc,0,Xc,Y_max)

--[[ TICS & GRID ]]--
```

```
if lblpos[1]==0 then
 Yl=Y_pixels-lblsize[1]
else
 Yl=Yc+1
end
if lblpos[2]==0 then
 Xl=2
else
 Xl=Xc+2
end
for xp=c[1]-math.floor((c[1]-xv[1])/tics[1])*tics[1],math.floor((xv[2]-c[1])/tics[1])*tics[1],tics[1]
do
 X=(xp-xv[1])*scale[1]
 draw.line(X,Yc-2,X,Yc+2)
 if grid then
  for Y=0,Y_pixels,4 do
   draw.pixel(X,Y)
  end
 end
 if lblsize[1]>0 and X>0 and X<X_pixels then
  lblxp=xp
  if math.abs(xp)<=lblcut then
   lblxp=0
```

```
 end
 draw.text(X+2,Yl,lblxp,1,lblsize[1])
 end
end
for yp=c[2]-math.floor((c[2]-yv[1])/tics[2])*tics[2],math.floor((yv[2]-c[2])/tics[2])*tics[2],tics[2]
do
 Y=(yv[2]-yp)*scale[2]
 draw.line(Xc-2,Y,Xc+2,Y)
 if grid then
  for X=0,X_pixels,4 do
   draw.pixel(X,Y)
  end
 end
 if lblsize[2]>0 and Y>0 and Y<Y_pixels then
  lblyp=yp
  if math.abs(yp)<=lblcut then
   lblyp=0
  end
  draw.text(Xl,Y-lblsize[2],lblyp,1,lblsize[2])
 end
end
end
```

```
function PlotPoint(X,Y,pointtype,pointsize)
--Non-filled circle:
if pointtype==0 then
 draw.point(X,Y,1,pointsize)
elseif pointtype==1 then
 draw.circle(X,Y,pointsize,1,1,-1)
 draw.point(X,Y)
--Crossed circle:
elseif pointtype==2 then
 draw.circle(X,Y,pointsize,1,1,-1)
 draw.line(X-pointsize,Y,X+pointsize,Y)
 draw.line(X,Y-pointsize,X,Y+pointsize)
--Filled circle:
elseif pointtype==3 then
 draw.circle(X,Y,pointsize,1,1,1)
--Non-filled rectangle:
elseif pointtype==4 then
 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,-1)
 draw.point(X,Y)
--Crossed rectangle:
elseif pointtype==5 then
 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,-1)
 draw.line(X-pointsize,Y,X+pointsize,Y)
```

```
 draw.line(X,Y-pointsize,X,Y+pointsize)

--Filled rectangle:

elseif pointtype==6 then

 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,1)

end

end


local function DrawLabel()

return

end


export{X_max=X_max,Y_max=Y_max,X_center=X_center,Y_center=Y_center,X_pixels=X_pixels,
Y_pixels=Y_pixels,SetScale=SetScale,Discontinuities=Discontinuities,PlotAxes=PlotAxes,PlotPoin
t=PlotPoint}
```

LNAutils/OrderMag

```
require("string")


function OrderMag(x)

local xexp=string.lower(string.format("%e",x))

local e=string.find(xexp,"e")

local n=string.len(xexp)

return 10^string.sub(xexp,e+1,n)
```

```
end


export{OrderMag=OrderMag}
```

LNAplot/PlotData

```
require ("table","draw","LNAplot/PlotUtil")


function PlotData(x,y,xv,yv,...)
--Optional arguments:
local wait=true
local ptype={1}
local psize={3}
local lwidth={0}
local tics={"auto","auto"}
local grid=true
local lblpos={0,0}
local lblsize={9,9}
local c="auto"
--
local points=#x
local scale
local sets,pws,lws,X,Xp,Y,Yp
```

```
if arg["n"]>=1 then
 wait=arg[1]
 if arg["n"]>=2 then
  ptype=arg[2]
  if type(ptype)=="number" then
   ptype={ptype}
  end
  if arg["n"]>=3 then
   psize=arg[3]
   if type(psize)=="number" then
    psize={psize}
   end
   if arg["n"]>=4 then
    lwidth=arg[4]
    if type(lwidth)=="number" then
     lwidth={lwidth}
    end
    if arg["n"]>=5 then
     tics=arg[5]
     if type(tics)~="table" then
      tics={tics,tics}
     end
     if arg["n"]>=6 then
```

```
    grid=arg[6]
    if arg["n"]>=7 then
     lblpos=arg[7]
     if type(lblpos)=="number" then
      lblpos={lblpos,lblpos}
     end
     if arg["n"]>=8 then
      lblsize=arg[8]
      if type(lblsize)=="number" then
       lblsize={lblsize,lblsize}
      end
      if arg["n"]>=9 then
       c=arg[9]
      end
     end
    end
   end
  end
 end
end
if type(y[1])=="number" then y={y} end
```

```
sets=#y
pws=#ptype
for i=pws+1,sets do
 ptype[i]=ptype[pws]
end
pws=#psize
for i=pws+1,sets do
 psize[i]=psize[pws]
end
lws=#lwidth
for i=lws+1,sets do
 lwidth[i]=lwidth[lws]
end
draw.onbuffer()
if xv[1]~=nil then
 scale=SetScale(xv,yv)
 PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)
else
 xv=yv[1]
 scale=yv[3]
 yv=yv[2]
end
Xp=(x[1]-xv[1])*scale[1]
```

```
Y={};Yp={}
for j=1,sets do
 Yp[j]=(yv[2]-y[j][1])*scale[2]
 PlotPoint(Xp,Yp[j],ptype[j],psize[j])
end
for i=2,points do
 X=(x[i]-xv[1])*scale[1]
 for j=1,sets do
  Y[j]=(yv[2]-y[j][i])*scale[2]
  if psize[j]>0 then
   PlotPoint(X,Y[j],ptype[j],psize[j])
  end
  if lwidth[j]>0 then
   draw.line(Xp,Yp[j],X,Y[j],1,lwidth[j])
   Yp[j]=Y[j]
  end
 end
 Xp=X
end
showgraph()
draw.update()
if wait then waitkey() end
showconsole()
```

```
return {xv,yv,scale}

end



export{PlotData=PlotData}
```