In the name of Allah,

Most Gracious, Most Merciful.

Article Title :

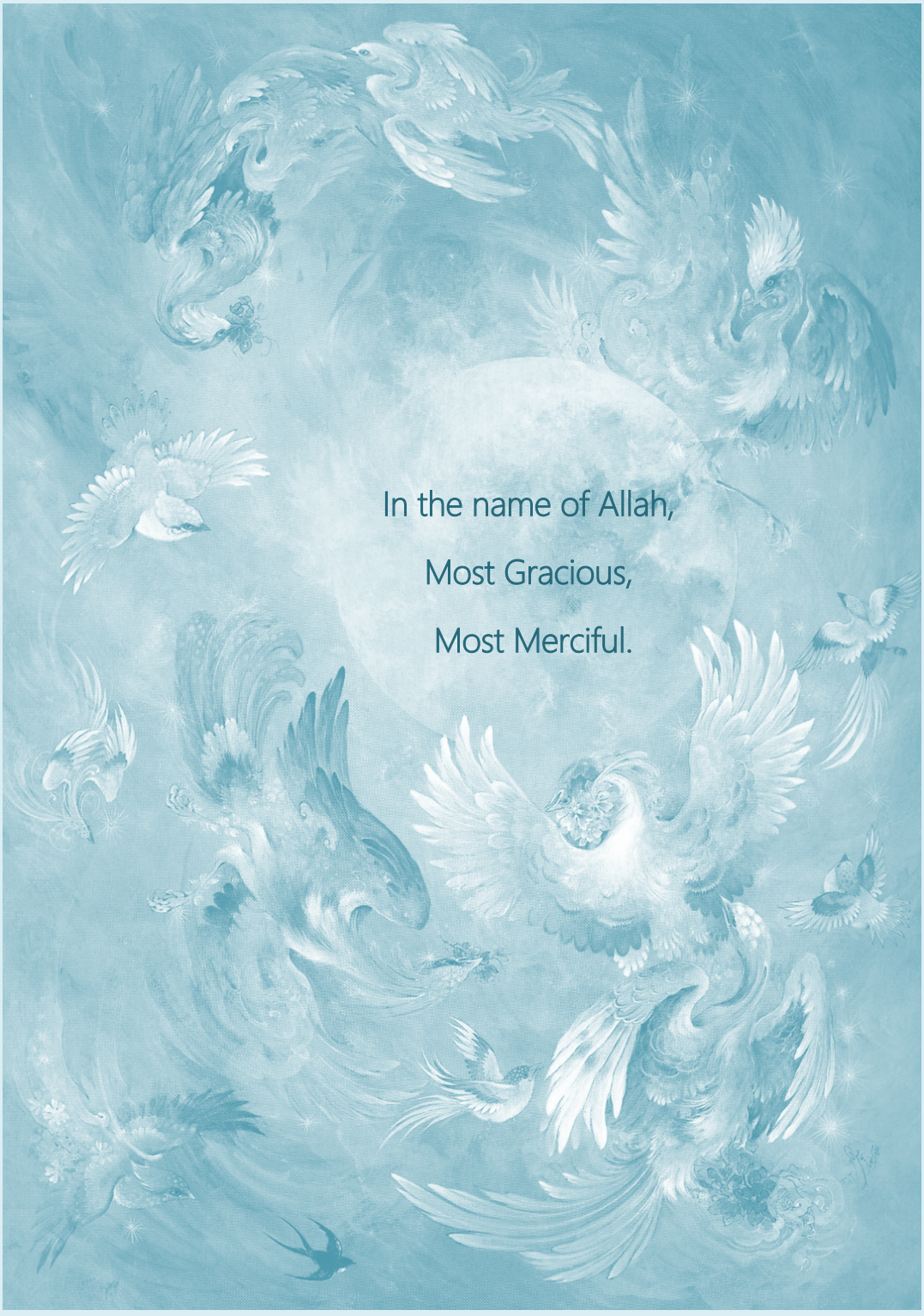
Miniatoric Infinity Plot And It,s Applications

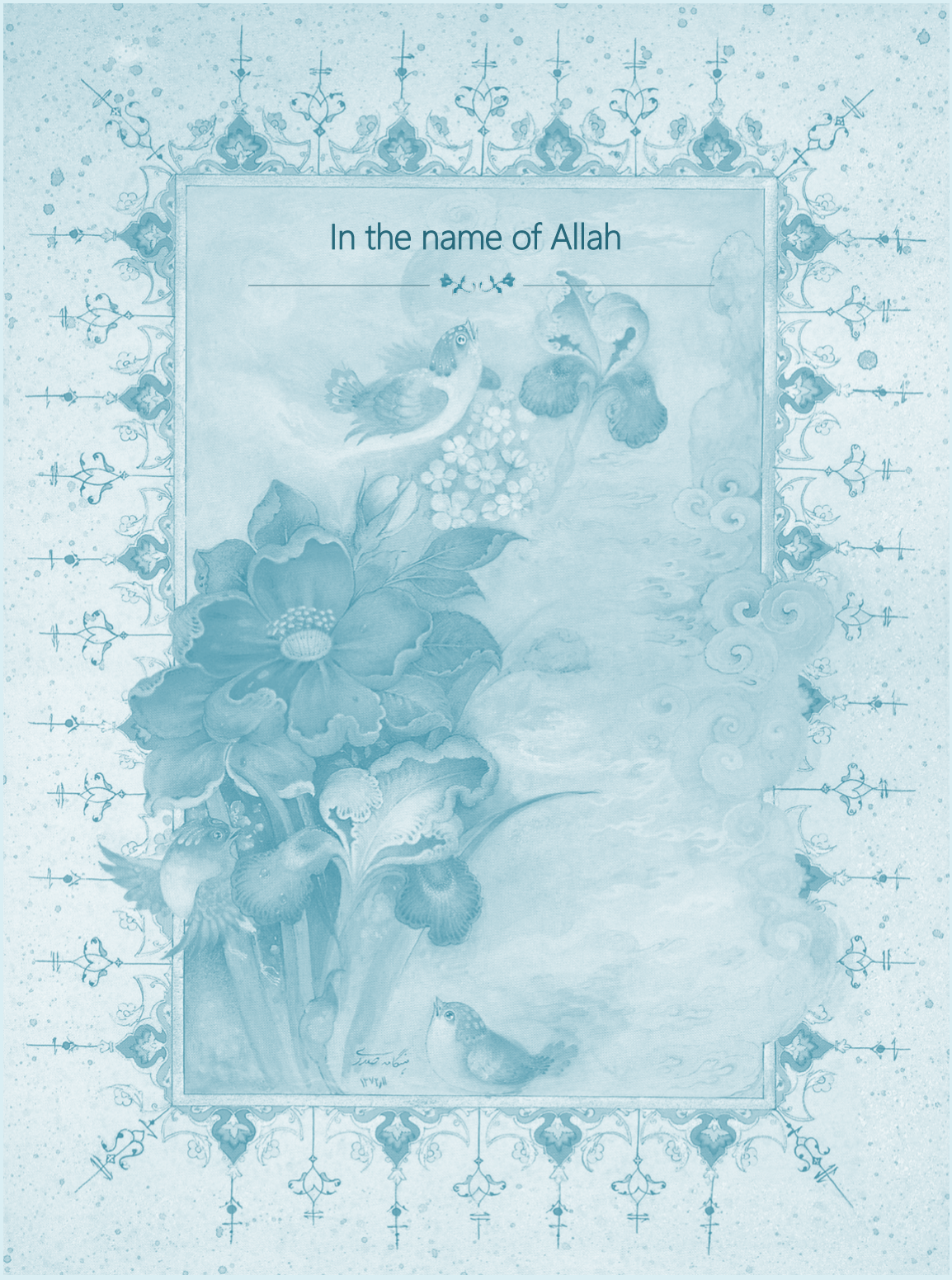
Digest Description Of Miniatoric Infinity Plot Concept :

- If either of the range end points of the horizontal range contains ±infinity, an infinity plot is generated.
- An infinity plot is obtained by transforming -infinity .. +infinity to $-\frac{\pi}{2}..\frac{\pi}{2}$ by a transformation that approximates arctan. This is a nice way of getting the entire picture of f(x) on the display.
- Such a graph, although distorted near x = -infinity and +infinity, contains a lot of information about the features of f(x).

The Article Is A Gift
From The GOD
To All The Believers In GOD

Do Not Forget The Ordained Prayer To GOD
Because Of This Great Manifest Signs Of The GOD.


In the name of Allah,
Most Gracious,
Most Merciful.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- ***BrentGhofrani Method to find roots***

*The function **BrentGhofrani** computes the root of a function within a given interval via the Brent Ghofrani method. This function combines the guaranteed convergence of the Bisection method and the speed of the Newton-Raphson method (which, however, may diverge). The **Brent Ghofrani** method is always converging, and it is usually much faster than Bisection method. Due to these advantages, it is considered as the method of choice for root finding. For convenience, the function **BrentGhofrani** has the same syntax as Bisect.*

Syntax

```
root = BrentGhofrani(ftrans,xl,xr,eps,maxit,show)
```

Returns the roots of the function **ftrans** that uses the transformed type of the function **f** transformed with the following formula**:**

```
ArcTan(f(Tan(x))
```

inside the interval **{xl,xr}** that can be within the interval **{-$\frac{\pi}{2}$, $\frac{\pi}{2}$}**. The arguments **eps , maxit** and **show** are optional. **eps** defines the desired accuracy (default: **Epsilon**, i.e., 1.12×10^{-16}); it can be set to ''auto'', which is equivalent to the default accuracy. **maxit** defines the maximum number of iterations (default: 100). **show** is a boolean argument that controls wether progress of the iterative process will be displayed or not (default: **false**); if set to **true**, the function value, **f(root)**, at each iteration will be displayed.

Example

The example program **XBrentGh** uses the function **BrentGhofrani** to find the root of the transformed function math.atan(f(math.tan(x))) where f is the function $e^{1-x} - \frac{x}{2}$ within the interval **[-$\frac{\pi}{2}$, $\frac{\pi}{2}$]**. In order to check the result, the program computes the function value at the root ontained, and shows the result graphically.
The Figure shows the results obtained by running this program. Note that function value **ftrans(root)** is too close to **zero**.

```
require("LNAplot/PlotFunc","LNAplot/PlotData","LNA/BrentGh")

local function f(x) return math.exp(1-x)-x/2 end
```

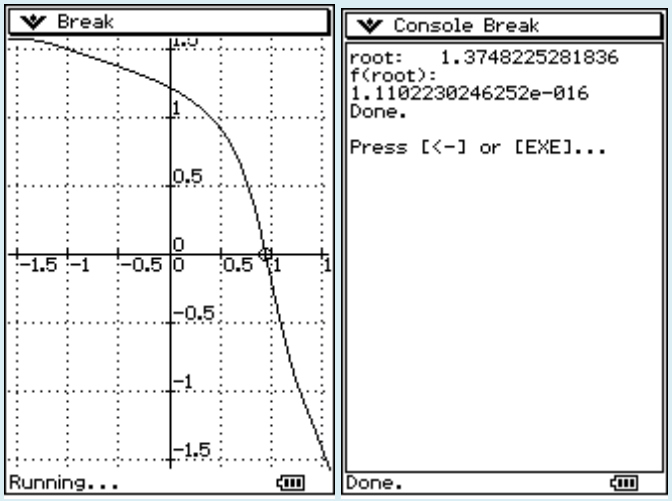
```
local function ftrans(x) return math.atan(f(math.tan(x))) end

local root,error,axesdata

root,error=BrentGhofrani(ftrans,-math.pi/2,math.pi/2)
print("root:",math.tan(root))
print("f(root):",f(math.tan(root)))

axesdata=PlotFunc(ftrans,{-math.pi/2,math.pi/2},{-math.pi/2,math.pi/2},false,1,"auto",true,1)
PlotData({root},{ftrans(root)},{},axesdata,true,2)
```

Example Program : **XBrentGH**



Results obtained by the example program **XBrentGH**

Remarks

The interval **{xl,xr}** should contain *exactly* one root. You may need to reduce the desired accuracy only in special cases. The optional argument **maxit** is rarly needed.
FILENAME: **BrentGH**.
DEPENDENCIES: **Epsilon**.

The Driver program code and it,s requirements code:

```
LNA/BrentGH
```

```
require("LNAutils/Epsilon")
```

```
local function NoRoot(fa,fb)
return (fa>0 and fb>0) or (fa<0 and fb<0)
end

function BrentGhofrani(f,xl,xr,...)
local tol=Epsilon
local maxit=100
local show=false
if arg["n"]>=1 then
 if arg[1]~="auto" then
 tol=arg[1] end
 if arg["n"]>=2 then
  maxit=arg[2]
  if arg["n"]>=3 then
   show=arg[3]
  end
 end
end
local root
local eps=Epsilon
local a,b,c,fa,fb,fc,d,e
   ,tol1,xm,p,q,r,s
if show then print(" i  discrepancy") end
a=xl;b=xr;fa=f(a);fb=f(b)
if NoRoot(fa,fb) then
 print("Error: Brent: Interval does not contain a root, or contains an even number of roots.")
 return
end
c=b;fc=fb
for i=1,maxit do
 if NoRoot(fb,fc) then
  c=a;fc=fa;d=b-a;e=d
 end
 if math.abs(fc)<math.abs(fb) then
  a=b;fa=fb;b=c;fb=fc
  c=a;fc=fa
 end
 tol1=2*eps*math.abs(b)+0.5*tol
 xm=0.5*(c-b)
 if show then printf("%2i  %+e\n",i,fb) end
 if (math.abs(xm)<=tol1)
```

```
or (fb==0) then
 root=b;return root
end
if (math.abs(e)>tol1)
and (math.abs(fa)>math.abs(fb)) then
 s=fb/fa
 if a==c then
  p=2*xm*s;q=1-s
 else
  q=fa/fc;r=fb/fc
  p=s*(2*xm*q*(q-r)
     -(b-a)*(r-1))
  q=(q-1)*(r-1)*(s-1)
 end
 if p>0 then q=-q end
 p=math.abs(p)
 if 2*p<math.min(3*xm*q-math.abs(tol1*q),math.abs(e*q)) then
  e=d;d=p/q
 else
  d=xm;e=d
 end
else
 d=xm;e=d
end
a=b;fa=fb
 if math.abs(d)>tol1 then
  b=b+d
 else
  b=b+math.sign(tol1,xm)
 end
 fb=f(b)
end
root=b
print("Warning: Brent: Maximum number of iterations reached, but the accuracy criterion is
not satisfied.")
return root
end

export{BrentGhofrani=BrentGhofrani}
```

LNAutils/Epsilon

```
Epsilon=1.12E-16
export{Epsilon=Epsilon}
```

The Example program code and it,s requirements code:

LNAutils/XBrentGh

```
require("LNAplot/PlotFunc","LNAplot/PlotData","LNA/BrentGh")


local function f(x) return math.exp(1-x)-x/2 end


local function ftrans(x) return math.atan(f(math.tan(x))) end


local root,error,axesdata


root,error=BrentGhofrani(ftrans,-math.pi/2,math.pi/2)

print("root:",math.tan(root))

print("ftrans(root):",ftrans(root))


axesdata=PlotFunc(ftrans,{-math.pi/2,math.pi/2},{-math.pi/2,math.pi/2},false,1,"auto",true,1)

PlotData({root},{ftrans(root)},{},axesdata,true,2)
```

LNAutils/PlotFunc

```
require ("table","draw","LNAplot/PlotUtil")
```

```
function PlotFunc(f,xv,yv,...)
--Optional arguments:
local wait=true
local lwidth={1}
local tics={"auto","auto"}
local grid=true
local lblpos={0,0}
local lblsize={9,9}
local c="auto"
local discont={}
--
local scale
local lws,funcs,x,xp,X,Xp,Y,Yp,disconts
local id={}
if arg["n"]>=1 then
 wait=arg[1]
 if arg["n"]>=2 then
  lwidth=arg[2]
  if type(lwidth)=="number" then
   lwidth={lwidth}
  end
  if arg["n"]>=3 then
```

```
tics=arg[3]
if type(tics)~="table" then
 tics={tics,tics}
end
if arg["n"]>=4 then
 grid=arg[4]
 if arg["n"]>=5 then
  lblpos=arg[5]
  if type(lblpos)=="number" then
   lblpos={lblpos,lblpos}
  end
  if arg["n"]>=6 then
   lblsize=arg[6]
   if type(lblsize)=="number" then
    lblsize={lblsize,lblsize}
   end
   if arg["n"]>=7 then
    c=arg[7]
    if arg["n"]>=8 then
     discont=arg[8]
    end
   end
  end
```

```
   end
  end
 end
end
end
lws=#lwidth
if type(f)=="function" then
 f={f}
end
funcs=#f
for i=lws+1,funcs do
 lwidth[i]=lwidth[lws]
end
discont,disconts=Discontinuities(funcs,discont)
for i=1,funcs do
 id[i]=1
 table.sort(discont[i])
end
draw.onbuffer()
if xv[1]~=nil then
 scale=SetScale(xv,yv)
 PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)
else
```

```
 xv=yv[1]
 scale=yv[3]
 yv=yv[2]
end
Xp=0;xp=xv[1]
Y={};Yp={}
for i=1,funcs do
 Yp[i]=(yv[2]-f[i](xv[1]))*scale[2]
end
for X=1,X_pixels do
 x=xv[1]+X/scale[1]
 for i=1,funcs do
  Y[i]=(yv[2]-f[i](x))*scale[2]
  if id[i]>disconts[i] or x<discont[i][id[i]] or xp>discont[i][id[i]] then
   draw.line(Xp,Yp[i],X,Y[i],1,lwidth[i])
  elseif xp<=discont[i][id[i]] and x>=discont[i][id[i]] then
   id[i]=id[i]+1
  end
  Yp[i]=Y[i]
 end
 xp=x;Xp=X
end
showgraph()
```

```
draw.update()

if wait then waitkey() end

showconsole()

return {xv,yv,scale}

end


export{PlotFunc=PlotFunc}
```

LNAutils/PlotUtil

```
require ("string","table","draw","LNAutils/OrderMag")


X_max=158

Y_max=213

X_center=79

Y_center=106

X_pixels=159

Y_pixels=214


local function SetScale(xv,yv)

local xscale,yscale

xscale=X_pixels/(xv[2]-xv[1])

yscale=Y_pixels/(yv[2]-yv[1])
```

```
return {xscale,yscale}
end


local function AutoXtics(range)
--tics=math.max(math.floor(range/7),1)
local tics=OrderMag(range)
if range<4*tics then
 tics=tics/2
end
return tics
end


local function AutoYtics(range)
--tics=math.max(math.floor((yv[2]-yv[1])/9),1)
local tics=OrderMag(range)
if range<4*tics then
 tics=tics/2
end
return tics
end


function Discontinuities(funcs,discont)
local D=table.copy(discont)
```

```
local id={}
local disconts={}
for i=1,funcs do
 if D[i]==nil then
  D[i]={}
 elseif type(D[i])=="number" then
  D[i]={D[i]}
 end
 disconts[i]=#D[i]
end
return D,disconts
end


function PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)
local Xc,Yc,X,Y,Xl,Yl,lblxp,lblyp
local lblcut=1E-8
--Auto tics selection:
if tics[1]=="auto" then
 tics[1]=AutoXtics(xv[2]-xv[1])
end
if tics[2]=="auto" then
 tics[2]=AutoYtics(yv[2]-yv[1])
end
```

```
--
if c=="auto" then
 c={0,0}
end
--[[ AXES ]]--
Xc=(c[1]-xv[1])*scale[1]
Yc=(yv[2]-c[2])*scale[2]
draw.line(0,Yc,X_max,Yc)
draw.line(Xc,0,Xc,Y_max)
--[[ TICS & GRID ]]--
if lblpos[1]==0 then
 Yl=Y_pixels-lblsize[1]
else
 Yl=Yc+1
end
if lblpos[2]==0 then
 Xl=2
else
 Xl=Xc+2
end
for xp=c[1]-math.floor((c[1]-xv[1])/tics[1])*tics[1],math.floor((xv[2]-c[1])/tics[1])*tics[1],tics[1]
do
 X=(xp-xv[1])*scale[1]
```

```
 draw.line(X,Yc-2,X,Yc+2)
 if grid then
  for Y=0,Y_pixels,4 do
   draw.pixel(X,Y)
  end
 end
 if lblsize[1]>0 and X>0 and X<X_pixels then
  lblxp=xp
  if math.abs(xp)<=lblcut then
   lblxp=0
  end
  draw.text(X+2,Yl,lblxp,1,lblsize[1])
 end
end
for yp=c[2]-math.floor((c[2]-yv[1])/tics[2])*tics[2],math.floor((yv[2]-c[2])/tics[2])*tics[2],tics[2]
do
 Y=(yv[2]-yp)*scale[2]
 draw.line(Xc-2,Y,Xc+2,Y)
 if grid then
  for X=0,X_pixels,4 do
   draw.pixel(X,Y)
  end
 end
```

```
 if lblsize[2]>0 and Y>0 and Y<Y_pixels then
  lblyp=yp
  if math.abs(yp)<=lblcut then
   lblyp=0
  end
  draw.text(Xl,Y-lblsize[2],lblyp,1,lblsize[2])
 end
end
end


function PlotPoint(X,Y,pointtype,pointsize)
--Non-filled circle:
if pointtype==0 then
 draw.point(X,Y,1,pointsize)
elseif pointtype==1 then
 draw.circle(X,Y,pointsize,1,1,-1)
 draw.point(X,Y)
--Crossed circle:
elseif pointtype==2 then
 draw.circle(X,Y,pointsize,1,1,-1)
 draw.line(X-pointsize,Y,X+pointsize,Y)
 draw.line(X,Y-pointsize,X,Y+pointsize)
--Filled circle:
```

```
elseif pointtype==3 then
 draw.circle(X,Y,pointsize,1,1,1)
--Non-filled rectangle:
elseif pointtype==4 then
 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,-1)
 draw.point(X,Y)
--Crossed rectangle:
elseif pointtype==5 then
 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,-1)
 draw.line(X-pointsize,Y,X+pointsize,Y)
 draw.line(X,Y-pointsize,X,Y+pointsize)
--Filled rectangle:
elseif pointtype==6 then
 draw.rect(X-pointsize,Y-pointsize,X+pointsize,Y+pointsize,1,1,1)
end
end

local function DrawLabel()
return
end

export{X_max=X_max,Y_max=Y_max,X_center=X_center,Y_center=Y_center,X_pixels=X_pixels,
Y_pixels=Y_pixels,SetScale=SetScale,Discontinuities=Discontinuities,PlotAxes=PlotAxes,PlotPoin
t=PlotPoint}
```

LNAutils/OrderMag

```
require("string")


function OrderMag(x)
local xexp=string.lower(string.format("%e",x))
local e=string.find(xexp,"e")
local n=string.len(xexp)
return 10^string.sub(xexp,e+1,n)
end


export{OrderMag=OrderMag}
```

LNAplot/PlotData

```
require ("table","draw","LNAplot/PlotUtil")


function PlotData(x,y,xv,yv,...)
--Optional arguments:
local wait=true
local ptype={1}
local psize={3}
local lwidth={0}
```

```
local tics={"auto","auto"}
local grid=true
local lblpos={0,0}
local lblsize={9,9}
local c="auto"
--
local points=#x
local scale
local sets,pws,lws,X,Xp,Y,Yp
if arg["n"]>=1 then
 wait=arg[1]
 if arg["n"]>=2 then
  ptype=arg[2]
  if type(ptype)=="number" then
   ptype={ptype}
  end
  if arg["n"]>=3 then
   psize=arg[3]
   if type(psize)=="number" then
    psize={psize}
   end
   if arg["n"]>=4 then
    lwidth=arg[4]
```

```
if type(lwidth)=="number" then
 lwidth={lwidth}
end
if arg["n"]>=5 then
 tics=arg[5]
 if type(tics)~="table" then
  tics={tics,tics}
 end
 if arg["n"]>=6 then
  grid=arg[6]
  if arg["n"]>=7 then
   lblpos=arg[7]
   if type(lblpos)=="number" then
    lblpos={lblpos,lblpos}
   end
   if arg["n"]>=8 then
    lblsize=arg[8]
    if type(lblsize)=="number" then
     lblsize={lblsize,lblsize}
    end
    if arg["n"]>=9 then
     c=arg[9]
    end
```

```
      end
     end
    end
   end
  end
 end
end
if type(y[1])=="number" then y={y} end
sets=#y
pws=#ptype
for i=pws+1,sets do
 ptype[i]=ptype[pws]
end
pws=#psize
for i=pws+1,sets do
 psize[i]=psize[pws]
end
lws=#lwidth
for i=lws+1,sets do
 lwidth[i]=lwidth[lws]
end
draw.onbuffer()
```

```
if xv[1]~=nil then
 scale=SetScale(xv,yv)
 PlotAxes(xv,yv,scale,tics,grid,lblpos,lblsize,c)
else
 xv=yv[1]
 scale=yv[3]
 yv=yv[2]
end
Xp=(x[1]-xv[1])*scale[1]
Y={};Yp={}
for j=1,sets do
 Yp[j]=(yv[2]-y[j][1])*scale[2]
 PlotPoint(Xp,Yp[j],ptype[j],psize[j])
end
for i=2,points do
 X=(x[i]-xv[1])*scale[1]
 for j=1,sets do
  Y[j]=(yv[2]-y[j][i])*scale[2]
  if psize[j]>0 then
   PlotPoint(X,Y[j],ptype[j],psize[j])
  end
  if lwidth[j]>0 then
   draw.line(Xp,Yp[j],X,Y[j],1,lwidth[j])
```

```
  Yp[j]=Y[j]
 end
end
Xp=X
end
showgraph()
draw.update()
if wait then waitkey() end
showconsole()
return {xv,yv,scale}
end


export{PlotData=PlotData}
```